

## Classical Control Theory Implementation Of A Robust Control System On A Laboratory Oil Rig Model Based On Quantitative Feedback Theory (Qft) In Matlab/Simulink, For National Industrial Sustainable Development

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### Abstract

The report presents the design of a Proportional plus Derivative (PD) controller, with the laboratory servo Rig as the plant model that gives a robust tracking performance when applied, with the magnetic brakes at both extremes of its travel (i.e., servo fit with brake and servo fit without brake), corresponding to a system with and without noise disturbance signals respectively, using Classical multiple input single output (MISO) quantitative feedback theory (QFT) approach (Feng and Lozano, 1999). The servo Rig is an exact miniature model of an Oil Rig installation in the real physical world, and so a study/analysis and understanding of the model comparatively makes the Oil Rig infrastructure easily understood and accurately modelled [the model being represented by mathematical equations in either continuous time (t), discrete time (z), sampled data and delayed response or in complex frequency (s) domain] and controlled as a consequence. The design processes include: The use of data collection and model fitting programs provided, to establish a set of transfer functions that reasonably represent the behaviours observed on the actual Servo Rig. Establishing a suitable set of design criteria for the tracking performance of the Rigs. Designing a suitable Robust (in a tracking sense) controller. Testing the controller in simulation and on several Rigs. Various frequency response techniques and analysis and mathematical modelling (based on abstraction of real systems, using s-domain transfer functions) have been employed, which include the traditional (i.e., Bode) sensitivity function, Bode log-magnitude, and gain plots, Nichol's chart design criteria, allowable plant parameter variations, pole/zero placement, Nyquist M and N circles plots, construction of plant templates, PD compensation method, disturbance rejection and the synthesis of a pre-filter using the nominal loop tracking transfer function. At the end of our design and on comparison, the SIMULINK as well as the real-time (or online) models in MATLAB for the Oil Rig setup systematically gave out accurate and identical outputs (i.e., the two superimposed models on each other correctly fitted each other), suggesting a successful design of our controller. It also gave same order of mathematical equations representations, thus Classical Control approach provides a formal solution to control systems design. The investigation of the behaviour of the system used in this control laboratory is based entirely on the MATLAB (matrix laboratory) software program.

**Keywords:** QFT, Classical Control, Lab Servo Oil Rig, MISO, MATLAB, SIMULINK

### INTRODUCTION

Many practical systems are characterized by high uncertainty (Patil and Kothawale, 2011, Abbas and Reza, 2008) which makes it difficult to maintain good stability margins and performance properties for closed loop systems (Jadhav, Kadu and Parvat, 2012 and Frye and Colgren, 2015). There is, therefore, the need for proper control of these systems, in order that their performances do not drift into the region of instability

which is highly undesirable and catastrophic. Hence the need for design of control systems (plant, controller and filter) within specified limits or bounds (boundaries) in order to have desired and optimum performance. This report presents the design of a robust process (laboratory servo Rig setup) control system, having 2 degrees of freedoms feedback structure

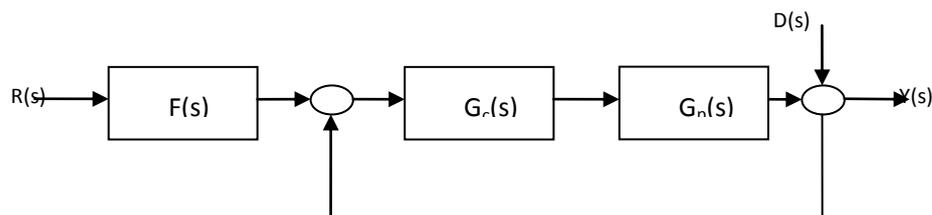


Fig 1. Multiple input single output (MISO) QFT Control Structure, having 2 degrees of freedom

The procedural design methodology used is the quantitative feedback theory (QFT), which is a classical frequency response based approach. It is a robust control method that typically involves a ‘worst-case’ design approach for a family of plants (representing some uncertainty) using a fixed controller. The QFT design methodology was originally developed for single input and single output linear time invariant (LTI) system with the design process, designed for improved sensitivity through the Nichols chart, in the given specified bands to particular variation of the plant. The controller is designed such that the time response of the closed loop system lies within acceptable limits for any plant parameters values within the specified range of variation.

**QFT DESIGN METHODOLOGY**

The first stage is the design of the plant model  $G_p(s)$ , the laboratory servo Rig. Using the servo Rig and data acquisition tool kit and arrangement in the laboratory, 2 sets of first order transfer functions in the s- domain (i.e., servo fit with brake and servo fit without brake) were obtained for the plant. These transfer functions effectively represented the specified bounds of the parameter variations in the plant model. They also represent the extremes or limits for worst case design. Next stage is the design of a proportional + derivative (PD) controller,  $G_c(s)$  using the QFT approach, first by arbitrarily choosing 2 sets of closed loop 2<sup>nd</sup> order transfer functions in s-domain for the signal envelop, and adjusting their  $k, \zeta$  and  $\omega_o$  such that the systems step response lies acceptably within the 2 sets, in a minimum phase system. The 2 sets are known as the upper and lower limits or bounds. This is achieved by letting:

$$|T(j\omega)|_{min} \leq |T(j\omega)| \leq |T(j\omega)|_{max}$$

Where,

$|T(j\omega)|_{min}, |T(j\omega)|_{max}$  are the magnitudes of the lower and upper limits/ boundaries transfer functions,

$|T(j\omega)|$  is the magnitude of the system step response.

The final design methodology is given below:

QFT design methodology-(Step By Step):

The step by step approach of a robust control design using the QFT is outlined below:

1. Synthesis of tracking , disturbance, and stability models of the plant
2. Specification of plant parameter uncertainty( obtained from the plant model of our laboratory experiment, using the servo Rig and the data acquisition setup, simulated via MATLAB)
3. Generation of plant templates, from the plant parameter specification
4. Selection of nominal plant  $P_o(s)$
5. Determination of stability contour
6. Generation and integration of QFT bounds
7. Shaping of nominal loop  $L_o(s) = G(s) P_o(s)$
8. Pre-filter design
9. Analysis and validation (time domain validation)

The actual laboratory results and discussions are presented below:

**Synthesis of Tracking, Disturbance, and Stability Models of The Plant**

The diagram below is the data collection and servo fit model, used for the plant model(the servo Rig, or motor)at both extremes of the magnetic brake travel (i.e., servo fit with brake and servo fit without brake), to obtain two sets of first order transfer functions, from which our **a** and **k** values are obtained

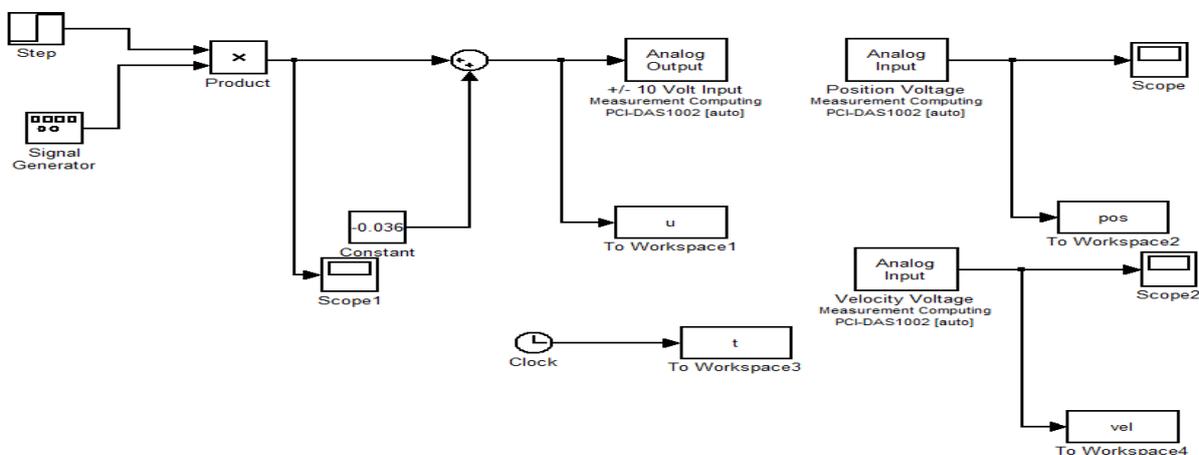


Fig. 2 data collection and servo fit model

**Servo fit model in simulation connection (Without brake, gain=3.66)**

The above Servo\_fit model in simulation mode, or connection without brake (gain=3.66) is shown below (fig. 3a). Also the waveforms after model fitting(i.e,

adjustment of the gain, time constant and k) are given below (fig. 3b):

Transfer function=  $0.73/0.12s + 1$

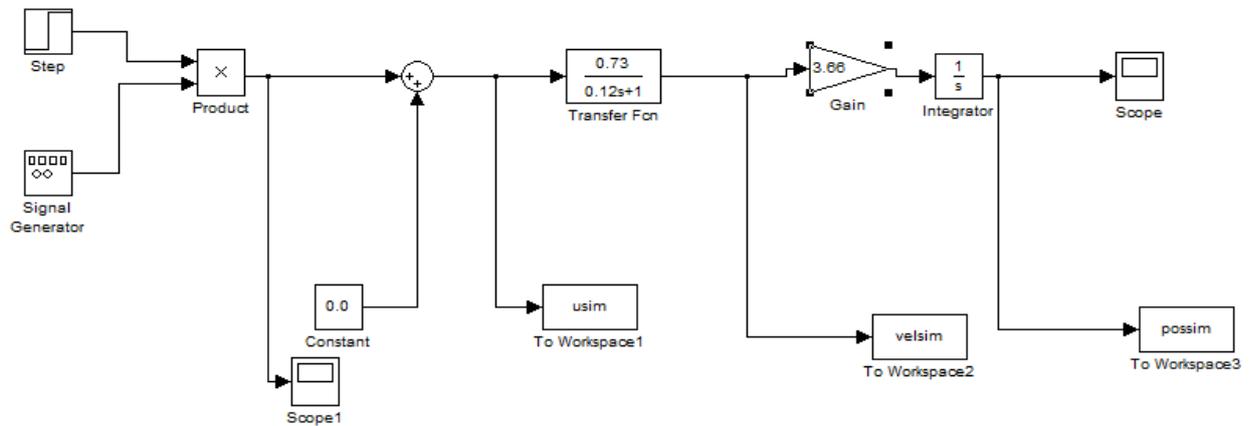


Fig. 3a

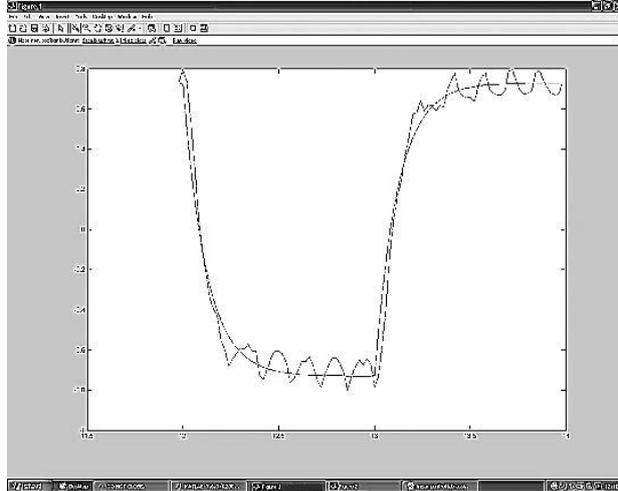


Fig. 3b(i)

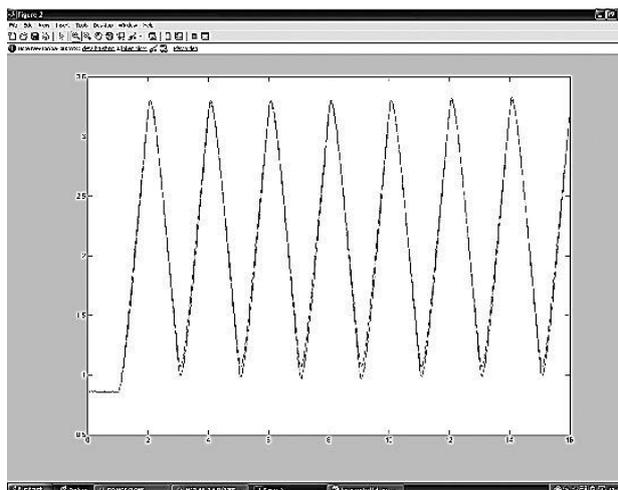


fig. 3b(ii)

Fig. 2. Servo fit model in simulation (without brake), showing the actual connection (fig 3a) and the wave outputs obtained (fig 3b).

**Servo fit model in simulation connection (With brake, gain=3.2)**

Transfer function=  $0.47/0.08s + 1$

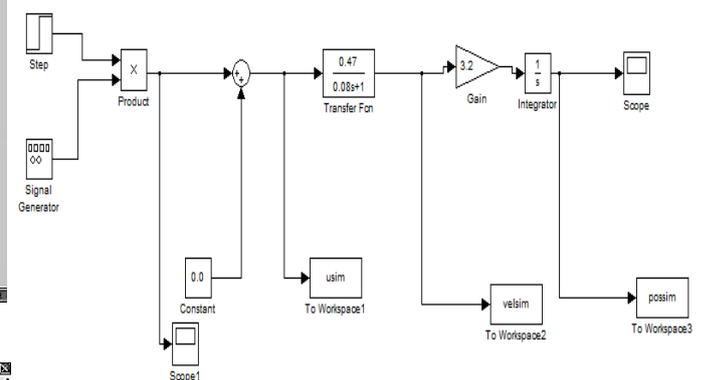


Fig. 4a

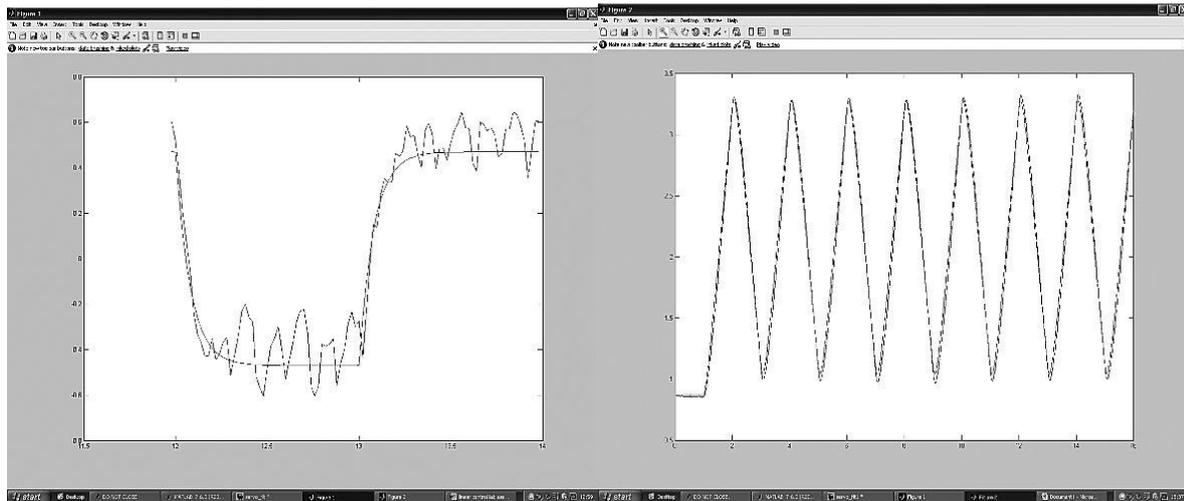


Fig 4b (i)

fig. 4b (ii)

Fig. 4 Servo fit model in simulation (with brake), showing the actual connection (fig 4a) and the wave outputs obtained (fig 4b)

**Specification of plant parameter uncertainty**

The time response of the system’s maximum allowable limits on the permitted variations of plant parameters  $\kappa, \xi$  and  $\omega_o$  (based on our arbitrary choice of a suitable upper ( $gu$ ) and lower ( $gl$ ) limits/bounds, for which our system step response should lie within), is chosen arbitrarily and is given below:

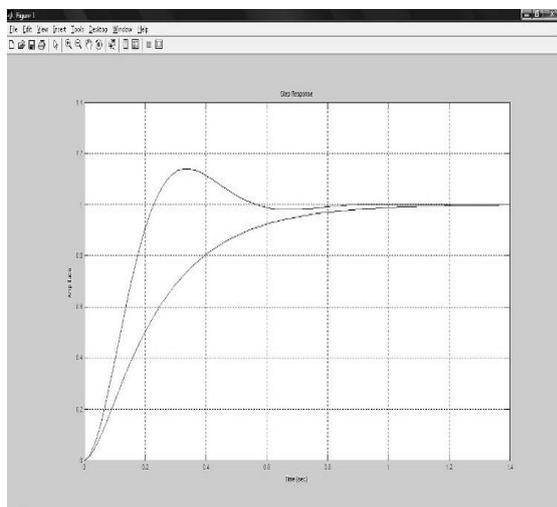


Fig. 5 upper and lower bounds ( $gu$  and  $gl$ ) step responses in frequency or s- domain

Note

$$:(gu = 121/s^2 + 11.66s + 121, gl = 100/s^2 + 26s + 100)$$

For  $gu: \xi = 0.53$ , (under damped nature)  $\omega_o = 11$ . Hence the maximum peak value  $Mp$ , and the peak time,  $tp$  are shown on the step response envelop below:

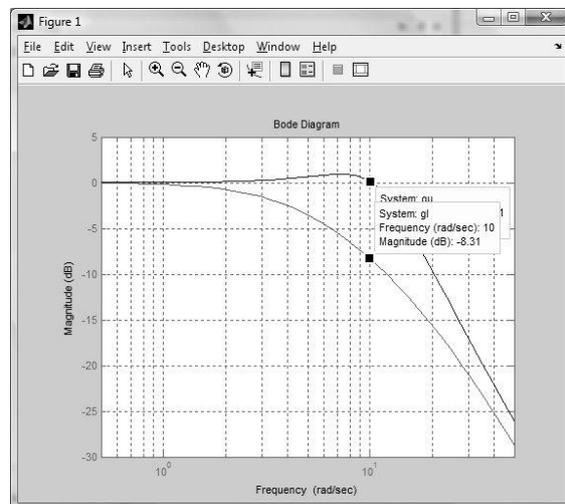


Fig. 6 the equivalent Bode logarithmic- magnitude plot

Fig. 6 shows the equivalent variation in closed loop log-magnitude response (Bode plot). Note that at a frequency of 10 rad/sec

$$\Delta \omega_o = 0.0646 - (-8.31) = 8.3746 \text{ dB}$$

Allowable variations in  $\log|T(j\omega_i)|$  ( $\Delta\omega_i$ ) in close-loop performance for this design are given below

Table 1. A table of selected frequencies and their corresponding variations in dB

$\omega_i$ (rad./sec.)	$\Delta\omega_i$ (dB)s
2.5	1.35
5	4.211
10	8.3746
15	13.592
25	19.895

**Generation of plant templates, from the plant parameter specification and Selection of Nominal Plant**

Collection of frequency responses is called templates. Quantitative feedback theory translates the known frequency responses and the desired performance specifications into required conditions (Chen and Balance, 1998) on the face and magnitude of the nominal loop L O.

The creation of templates for unknown plants in question is the first step in Quantitative Feedback Theory. For plants with a general uncertain structure, QFT is used to identify crucial interior points in the uncertain parameters which map the boundaries of the plant templates presented. It shows how the boundaries of the templates are only generated by the edges of the uncertain parameters. (Chen and Balance, 1998)

Nichols chart (NC) templates characterizing the variations of the plant uncertainty for the selected frequencies of 2.5, 5, 10, 15 and 25 rad/sec. Their corresponding gains (in dB, shown in table 1) and phase in degrees are computed and plotted on the Nichol’s Chart, in order to determine the closed-loop sensitivity boundaries.

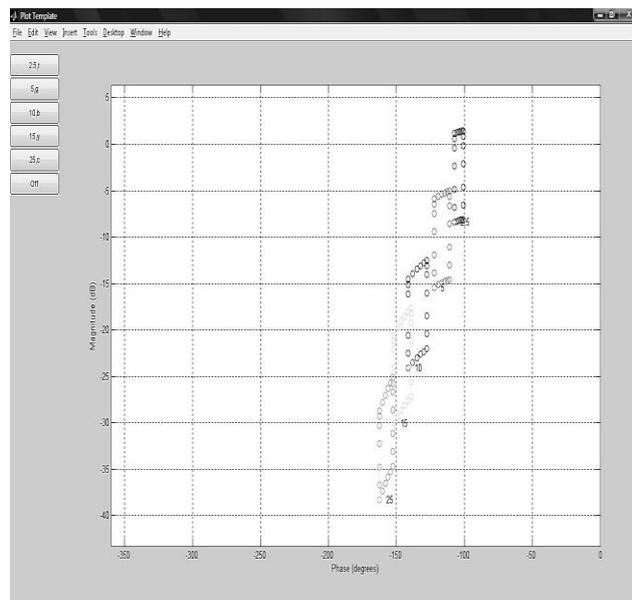


Fig. 7 NC plant templates showing the range of plant parameter variations for the selected  $\omega_i$

**Determination of stability contour, Generation and integration of QFT bounds and Shaping of nominal loop  $L_o(s)=G(s) P_o(s)$**

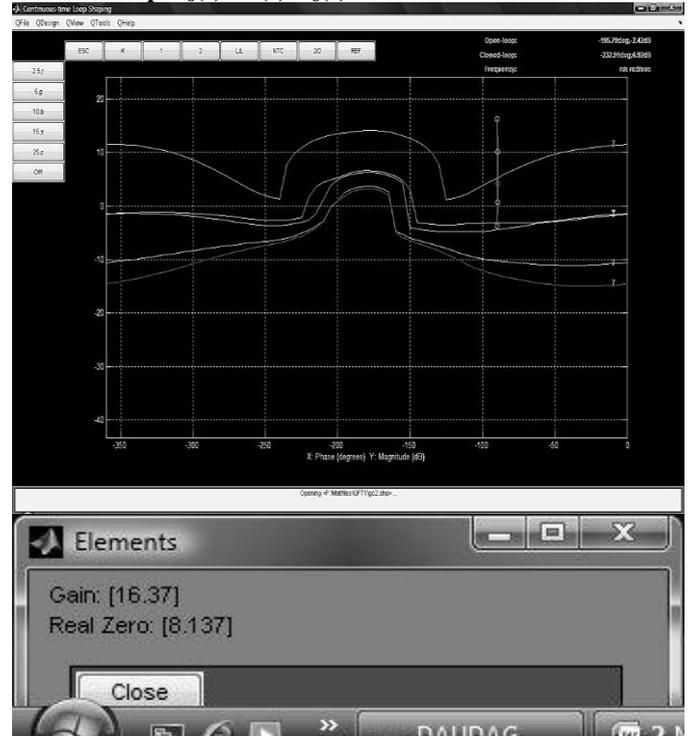


Fig. 8, showing the continuous time loop shaping, for the plant templates.

**Loop shaping:** Quantitative feedback theory involves the design of a nominal loop function that meets its bounds. The nominal loop is the product of the nominal plant of the controller (Borghesani,Chait and Yaniv, 2003). The nominal loop has satisfied the worst case of all bounds (Zolotas and Halikias, 1998). Nominal loop shaping is done using lp shape with the generic call:

```
lpshape (w1,ubdb,p0,c0,phs);
```

Notice the line and the circles with different colours on it. Each circle must be above its respective colour, if the design of a plant that can be controllable is to be realized/ achieved. The system gain is 16.37dB and contains a real zero 8.137

**Pre-filter design**

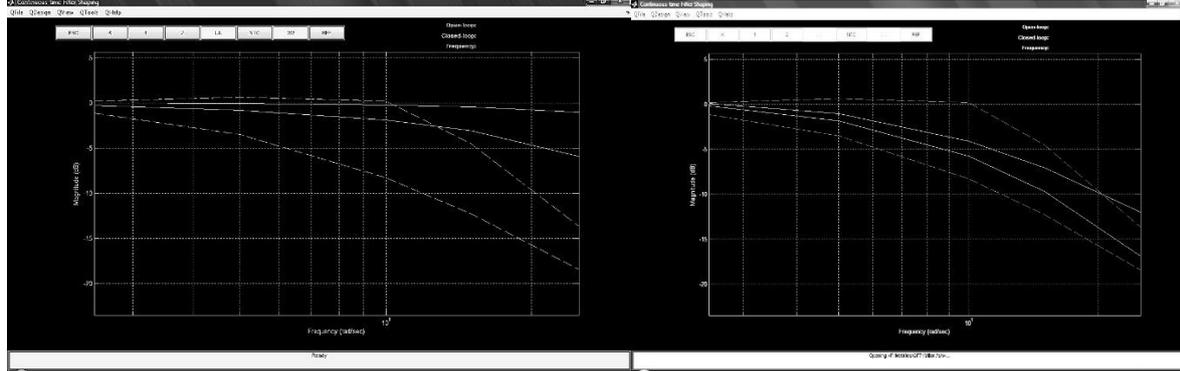


Fig.9a

Fig.9b (i)



Fig.9b (ii)

Fig 9. Showing the continuous time loop shaping filter for the pre-filter stage (fig 9a) before adjusting the gain of the first-order closed-loop frequency response (fig 9b), after the adjustment. The gain is 1.066dB, with 3 real poles: 7.73, 68.46 and 86.46

**Analysis and validation (time domain validation)**

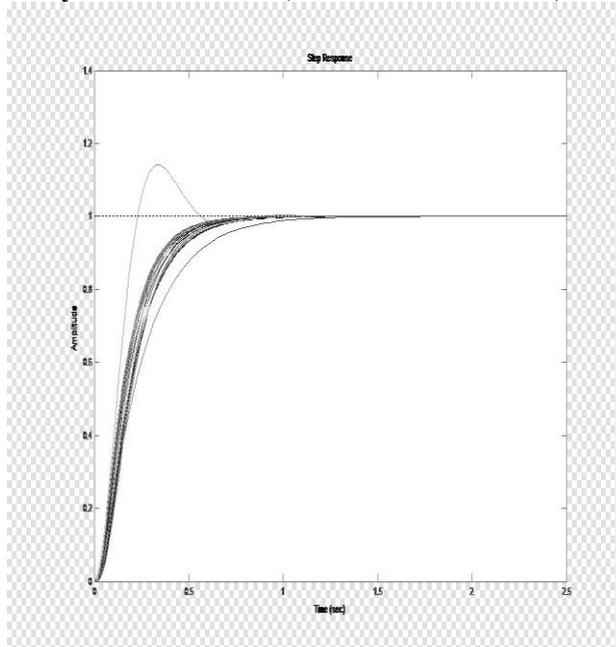


Fig.10 The final result for the time response for the controller design, showing all the stages of the design, with the bounds clearly noticed

*Combining with the bound:* We have computed bounds for all performance and margin problems. The next step is to combine them all (i.e. the bounds) into a single variable. Fig. 10 depicts it in the program. The program run/command is:

```
bdb = grpbnds (bdb1, bdb2, bdb3);
```

It is always simpler to work with a single variable.

Plotbnds (bdb)

The family of plants for which our Oil Rig belongs is shown in fig. 10, lying between the bounds of the controller, all lying within the upper and lower bounds of the unit step response (shown by the upper green line and the lower purple colour) Qft simulation without brake

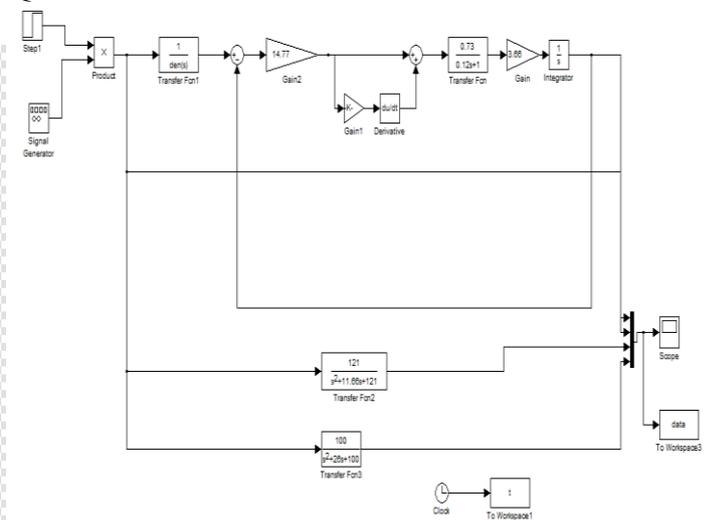


Fig.11 (a)

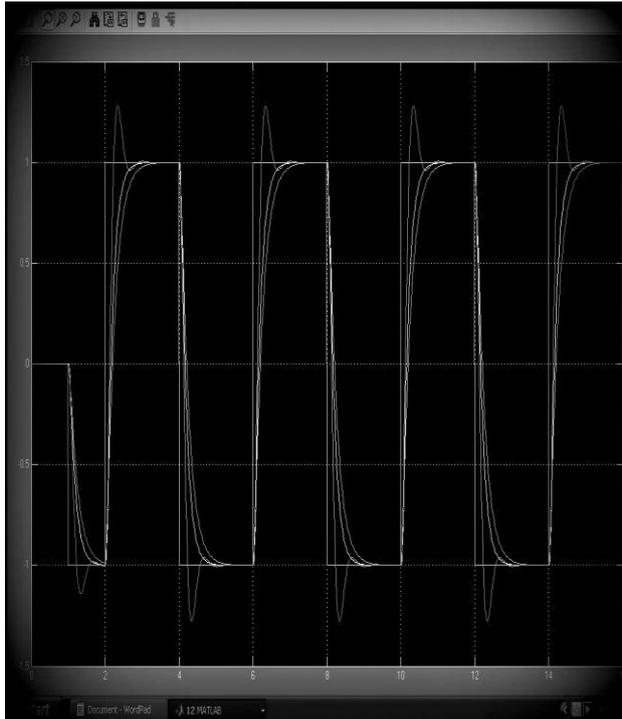


Fig. 11 (b)  
Fig.11 QFT simulation without brake (a) connection for the simulation (b) output waveform

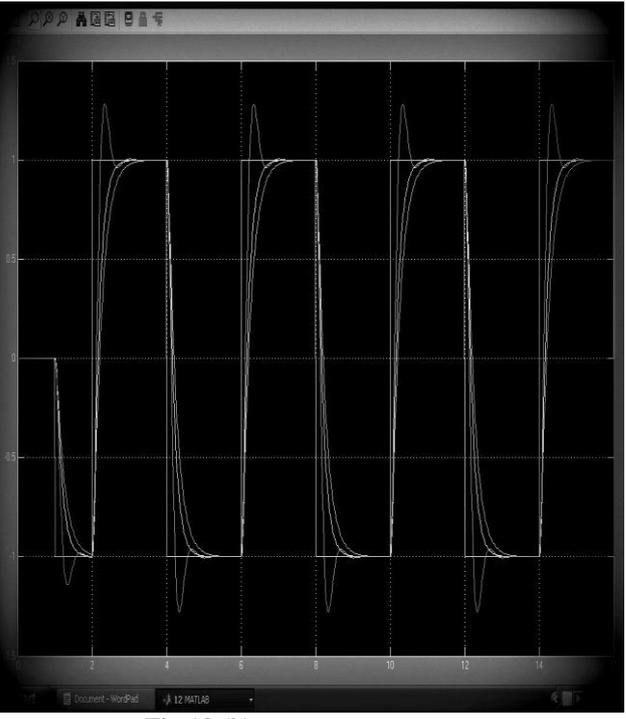


Fig.12 (b)  
Fig. 12 QFT simulation with brake (a) connection for the simulation (b) output waveform

**Qft simulation with brake**

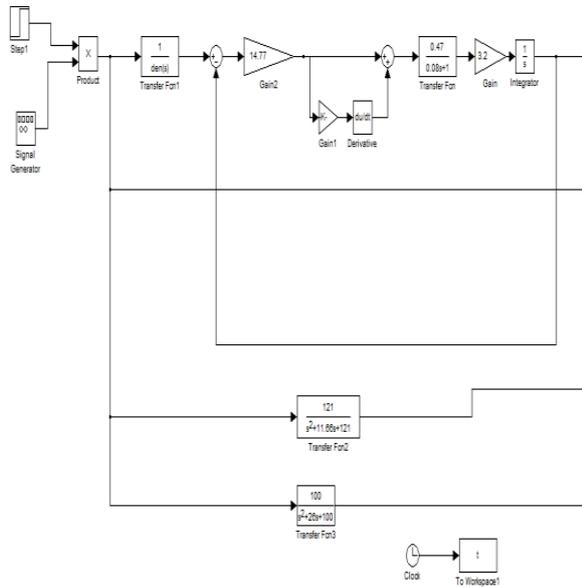


Fig. 12(a)

**Final design of the complete control system**

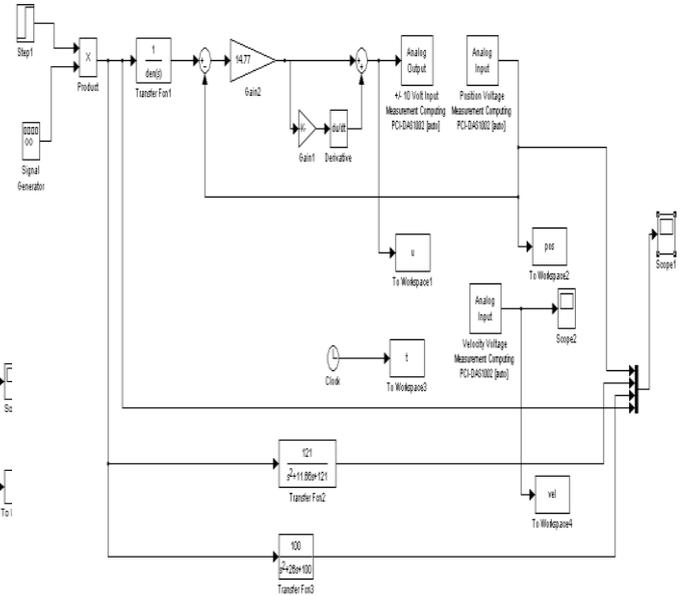


Fig. 13a (i)

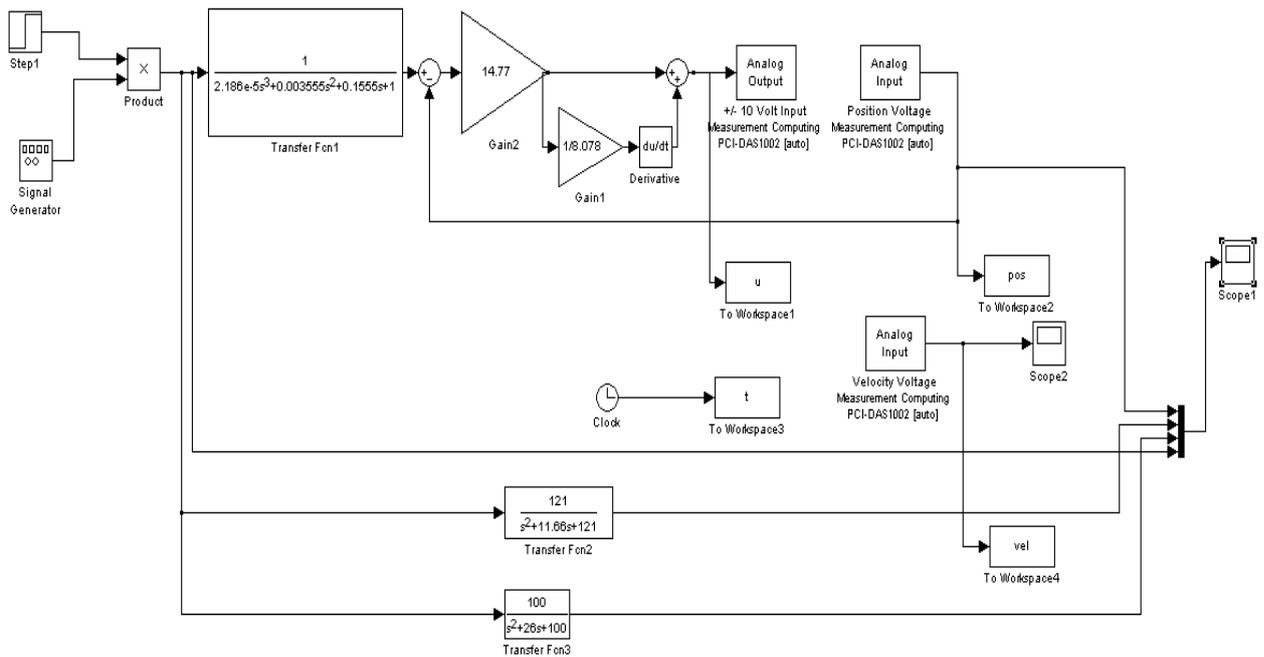


Fig. 13a (ii)

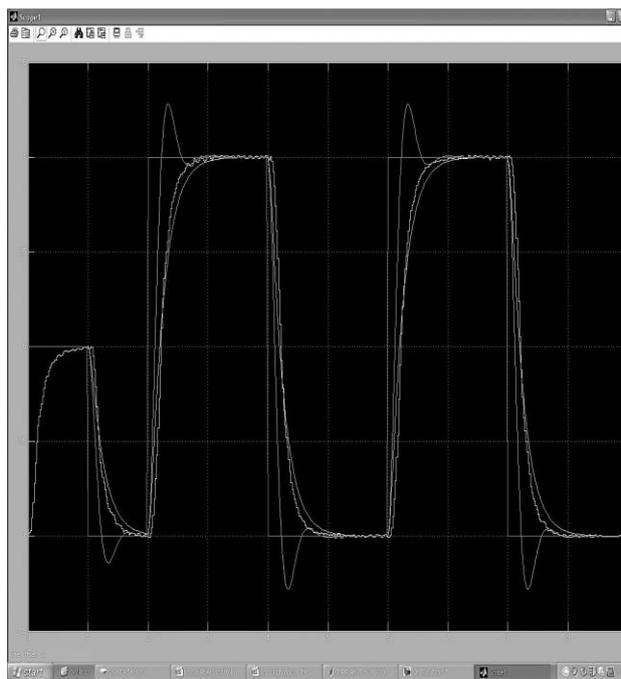


Fig. 13(b)

Fig. 13 (a) shows the actual final circuit connection in MATLAB/SIMULINK for the complete control system design, and fig. 13(b) shows the online (or real time) simulation of the complete control system. The middle line is the controller waveform signal while the upper (seen overshooting) and lower lines are the waveforms for the upper and lower bounds *gu and gl* respectively.

### Limitation of the Study

The study is limited by test rig used (being only a model or prototype) and laboratory data generated, however, its application goes beyond the laboratory. It should be noted that the aim is the development of the process of designing a real life controller for the Manufacturing, Aerospace, Robotics and Process industries, for example, for industrial development and sustainability.

### CONCLUSION

The waveforms obtained using QFT for the same input signals have the same pattern. Save for some negligible or insignificant error coefficients present as disturbance or spurious noise covariance signals which are bound to exist alongside the set-point or reference input signal in any control system design (the spurious signals can be seen as glitches on the controller signal/ waveform), we would have obtained exactly identical/similar waveforms signals for a (unit) step input signal to the control system designed, in simulation [fig.12 (b) ] and in real-time [fig. 13(b)]. We obtained the model for our filter/pre-filter to be

$\frac{1}{(2.186e^{-5}s^3 + 0.003555s^2 + 0.1555s + 1)}$  which a 3rd order system is. With a controller gain of 14.77 dB, the plant model for the **brake** and **no-brake** modifications of the parameter estimates are  $\frac{0.47}{0.08s + 1}$  with a gain of 3.2dB and  $\frac{0.73}{0.12s + 1}$  with a gain of 3.06dB respectively. The plant is designed within the specified upper and lower bounds/limits of

$$g_u = \frac{121}{(s^2 + 11.66s + 121)} \quad \text{and}$$

$$g_l = \frac{100}{(s^2 + 26s + 100)} \quad \text{respectively,}$$

i.e.,  $|T(j\omega)|_{\min} \leq |T(j\omega)| \leq |T(j\omega)|_{\max}$ , in which we see that  $T(j\omega)_{\min} = \frac{100}{(s^2 + 26s + 100)}$ ,  $T(j\omega)_{\max} = \frac{121}{(s^2 + 11.66s + 121)}$  and

$$T(j\omega) = \frac{1}{(2.186e^{-5}s^3 + 0.003555s^2 + 0.1555s + 1)}$$

is the overall system step response. From MATLAB on-line simulations results, the system step response transfer function in s-domain is a non-minimum phase system corresponding to a dynamic system response with a gain of 1.066dB, with 3 real poles: 7.73, 68.46 and 86.46. Since the system poles, being the roots of our characteristic equation, are real and negative [i.e. all the triple roots lying on the negative half of the complex s-plane as the characteristic equation is of the form  $(s - 7.73)(s - 68.46)(s - 86.46)$ ], a condition for stability, then it is a stable system. Stability was optimized using the continuous time loop shaping filter of the Nichols Chart plant templates as well as the gain adjustment of the pre-filter in which critical values should be avoided. The bounds represent the region within which the controller can function/operate in a stable and controller manner. Outside this region, the controller will be unstable, malfunction and rotate abnormally and uncontrollably. This is highly undesirable and dangerous (catastrophic) and so must be prevented by all means and at all cost.

Hence we have designed a Robust control system in terms of performance, and capable of self-sustaining.

#### APPLICATION

A practical application of this design is a **real life** Oil Rig installation of a remote outstation of any multinational oil company in Nigeria like Shell Petroleum Development Company (SPDC), ExxonMobil Nigeria unlimited, Chevron Nigeria or the Nigerian Liquefied Natural Gas (NLNG) and the Nigerian National Petroleum Corporation (NNPC), whose oil rig installation is newly installed and needs a control system to control its plant (i.e. an Actuator and Process), or an old installation that is not working in synchronism with its controller and so would need to be tuned by re-modelling, in order to function and operate effectively and properly within the specified boundaries/regions of stable and controllable operation. This is but one example out of the countless that exist where control system design is employed to advantage.

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