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APPLICATION OF TWO STEP CONTINUOUS HYBRID BUTCHER'S METHOD IN BLOCK FORM FOR THE SOLUTION OF FIRST ORDER INITIAL VALUE PROBLEM

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ABSTRACT

The two steps Hybrid Butcher's Method was reformulated for applications in the continuous form. The process produces some schemes which were combined in order to form an accurate and efficient block method for solution of ordinary differential equations (Ode's). The suggested approach eliminates requirements for a starting value and its speed proved to be up when computations with the Block Discrete schemes were used. The order of accuracy and stability of the block method is discussed and its accuracy established numerically.

Keywords: Hybrid Butcher's (HBM) Block Method; Region of Absolute Stability (RAS); Multistep Collocation (MC)

INTRODUCTION

In relevant literatures, conventional linear multistep methods including hybrid ones have been made continuous through the idea of Multistep Collocation (MC) [15-19]. The continuous multistep method (CMM) produces piece-wise polynomial solutions over k-steps $[x_n, x_{n+k}]$ for the first order ODE's. Of note is that the implicit (CMM) interpolant is not to be directly used as the numerical integrator, but the resulting discrete multistep schemes which is derived from it, which will now be self-starting and can be applied for solutions of initial value problems. In this paper, we developed a two-step Hybrid Butcher's Method in Block form for solution of first order initial value problems.

The analysis of our method and its application to numerical problems, not only proved its efficiency but its accuracy as well.

DERIVATION OF THE METHOD

Consider the initial value problem for the ordinary differential equation of the form:

$$y'(x) = f(x,y) \quad , \quad y(0) = y_o, \quad a \le x \le b \tag{1}$$

The general linear k-step LMM for (1) is given by the difference equation

$$\sum_{j=o}^{k} \propto_j y_{n+j} = h \sum_{j=o}^{k} \beta_j y_{n+j} \tag{2}$$

Where $\propto_j$ and β_j are real coefficients α_o, β_o not both zero with $\alpha_k = 1$

General Multistep Collocation (MC) linked to Continuous Multistep Method (CMM)
Let us first give a general description for the method of multistep collocation (MC) and its link to continuous Multistep Method (CMM) for (1). In equation (1), f is given and y is sought as

$$y = a_1\emptyset_1 + a_2\emptyset_2 + \cdots + a_p\emptyset_p \ldots \tag{3}$$

Where

$a = (a_1, a_2, \dots, a_p)^T$ and $\emptyset = (\emptyset_1, \emptyset_2 \dots \emptyset_p)^T$

$x_n \le x \le_{n+k}$, where $n = 0, k, \dots, n-k$ and T denote transpose of.

Equation (2) can be re-written as

$$y = (a_1, a_2, \dots, a_p)^T\, (\emptyset_1, \emptyset_2 \dots \emptyset_p)^T \quad (4)$$

The unknown coefficients $a_1, a_2, \dots, a_n$ are determined using respectively the $r(0 < r \le k)$ interpolation conditions and the $s > 0$ distinct collocation conditions, $p = r + s$ as follows

$$\sum_{i=1}^{p} a_j\, \emptyset_j(x_i) = y_i \quad ,(i = 1, \dots, r)$$
$$\sum_{i=1}^{p} a_j\, \emptyset'_j(x_i) = f_i\,, \quad (i = 1, \dots s) \quad (5)$$

This is a system of P linear equations from which we can compute values for the unknown coefficients provided (5) is assumed non-singular, for the distinct points x_i and c_i non-singular system is guaranteed (see proof in Yusuph and Onumanyi (2002). We can write (5) as a single set of linear equations of the form

$D\underline{a} = F$

$$\underline{a} = \underline{D}^{-1}\underline{F} = \underline{CF} \quad (6)$$

Where, $\underline{F} = (y_1, y_2, \dots, y_r\,,\ f_1\,,\ f_2, \ \dots\ f_s\,)^{\mathrm{T}}$ (7)

Substituting the vector $\underline{a}$, given by (6) and $\underline{F}$ by (7) into (4) gives

$$y = (y_1, y_2, \dots, y_r\,,\ f_1\,,\ f_2, \ \dots\ f_s) C^T (\emptyset_1\ ,\ \emptyset_2, \dots, \emptyset_p)^T \quad (8)$$

Equation (8) is the continuous MC interpolant C^T known explicitly in the form

$$\begin{pmatrix} C_{11} & C_{12} & C_{1p} \\ C_{21} & C_{22} & C_{2p} \\ C_{r1} & C_{r2} & C_{rp} \\ C_{p1} & C_{p2} & C_{pp} \end{pmatrix} \begin{pmatrix} \emptyset_1 \\ \emptyset_2 \\ \emptyset_r \\ \emptyset_p \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{p} C_{j1}\, \emptyset_j \\ \sum_{j=1}^{p} c_{j2}\, \emptyset_j \\ \sum_{j=1}^{p} C_{jr+1}\, \emptyset_j \\ \sum_{j=1}^{p} C_{jp}\, \emptyset_j \end{pmatrix} \begin{pmatrix} \propto_1 \\ \propto_2 \\ \beta_1 \\ \beta_s \end{pmatrix} \quad (9)$$

$$F^T C^T \emptyset = (\propto_1 y_1 + \propto_2 y_2 + \dots + \propto_r y_r + \beta_1 f_1 + \beta_2 f_2 + \dots + \beta_s f_s)$$

Or

$$F^T C^T \emptyset = \sum_{j=1}^{r} \propto_j y_j + h_j \left(\sum_{j=1}^{s} {\beta_j}/{h_j}\, f_j \right) \quad (10)$$

Where from (9)

$$\propto_j = \sum_{j=1}^{r} C_{qi}\ \emptyset_j \quad , \quad j = 1\ , \dots ,\ r$$
$$\beta_j / h_j = \sum_{q=1}^{p} \left[\frac{C_{qi+r}}{h_i}\right] \emptyset_j \quad , \quad j = 1\ , \dots ,\ s \quad (11)$$

Therefore

$$y = \sum_{j=1}^{r} \propto_j y_j + h_j \left[\sum_{j=1}^{s} {}^{\beta_j}/_{h_j} \right] f_j \qquad (12)$$

Where $\propto_j$, ${}^{\beta_j}/_{h_j}$ are given by (11). Hence (12) with (11) is the CMM interpolant with uniform or variable step-size.

DERIVATION OF PROPOSED METHOD

We proposed an approximate solution to (1) in the form

$$y_p(x) = \sum_{j=o}^{s+r-1} a_j x^i, i = 0\,(1)(s + r - 1) \qquad (13)$$

With $s = 4$, $r = 2$ and $p = s + r - 1$, also $\propto_j, \beta_j$, $j = 0, 1, (s + r - 1)$ are the parameters to be determined, where p is the degree of the polynomial interpolant of our choice.

Specifically, we interpolate equation (13) at $\{x_{n+2}, x_{n+3/2}, x_{n+4/3}, x_{n+5/3}, x_{n+5/4}, x_{n+7/4}\}$ and collocate (13) at $x_{n+4/3}$ and obtained a continuous form for the solution $\overline{y}(x) = VC^T P(x)$ from the system of the equation in the matrix below.

The general form of the new method is expressed as:

$$y(x) = \alpha_0 y_n + \alpha_1 y_{n+1} + h[\beta_0 f_n + \beta_1 f_{n+1} + \beta_{n+3/2} f_{n+3/2} + \beta_2 f_{n+2}] \qquad (14)$$

The matrix D of the new method expressed as:

$$\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 \\ 0 & 1 & 2x_{n+3/2} & 3x_{n+3/2}^2 & 4x_{n+3/2}^3 & 5x_{n+3/2}^4 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 \end{pmatrix} \begin{pmatrix} \propto_0 \\ \propto_1 \\ \beta_0 \\ \beta_1 \\ \beta_{3/2} \\ \beta_2 \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+1} \\ f_n \\ f_{n+1} \\ f_{n+3/2} \\ f_{n+2} \end{pmatrix} \qquad (15)$$

Mathematical software is used to obtain the inverse of the matrix D in equation (15) were values for $\propto_{i's}, (i = 0, 1)$ and $\beta_{i's}, (i = 0, 1, {}^3/_2, 2)$ is established. After some manipulation to the inverse, we arrived at the continuous form of the solution as

$$\begin{aligned} q := & \left(-\frac{180}{31}\frac{\xi^2}{h^2} + \frac{260}{31}\frac{\xi^3}{h^3} - \frac{135}{31}\frac{\xi^4}{h^4} + 1 + \frac{24}{31}\frac{\xi^5}{h^5}\right) y_n + \Bigg(\\ & -\frac{260}{31}\frac{\xi^3}{h^3} + \frac{180}{31}\frac{\xi^2}{h^2} - \frac{24}{31}\frac{\xi^5}{h^5} + \frac{135}{31}\frac{\xi^4}{h^4}\Bigg) y_{n+1} + \Bigg(\xi \\ & - \frac{571}{372}\frac{\xi^4}{h^3} - \frac{1123}{372}\frac{\xi^2}{h} + \frac{613}{186}\frac{\xi^3}{h^2} + \frac{8}{31}\frac{\xi^5}{h^4}\Bigg) f_n + \Bigg(\\ & -\frac{142}{31}\frac{\xi^4}{h^3} - \frac{117}{31}\frac{\xi^2}{h} + \frac{231}{31}\frac{\xi^3}{h^2} + \frac{28}{31}\frac{\xi^5}{h^4}\Bigg) f_{n+1} \\ & + \left(\frac{208}{93}\frac{\xi^4}{h^3} + \frac{112}{93}\frac{\xi^2}{h} - \frac{272}{93}\frac{\xi^3}{h^2} - \frac{16}{31}\frac{\xi^5}{h^4}\right) f_{n+\frac{3}{2}} + \Bigg(\\ & -\frac{59}{124}\frac{\xi^4}{h^3} - \frac{27}{124}\frac{\xi^2}{h} + \frac{35}{62}\frac{\xi^3}{h^2} + \frac{4}{31}\frac{\xi^5}{h^4}\Bigg) f_{n+2} \end{aligned} \qquad (16)$$

Evaluating (16) at x_{n+2}, $x_{n+3/2}$, $x_{n+4/3}$, $x_{n+5/3}$, $x_{n+5/4}$, and $x_{n+7/4}$ and its first derivative evaluated at $x = x_{n+4/3}$ yielded the following set of discrete schemes respectively.

$$31y_{n+2} - 32y_{n+1} + y_n = \frac{h}{3}\left[15f_{n+2} + 64f_{n+3/2} + 12f_{n+1} - f_n\right]$$
$$496y_{n+3/2} - 459y_{n+1} - 37y_n = \frac{3}{4}h\left[-9f_{n+2} + 160f_{n+3/2} + 216f_{n+1} + 13f_n\right]$$
$$2511y_{n+4/3} - 2368y_{n+1} - 143y_n = \frac{h}{3}\left[-68f_{n+2} + 768f_{n+3/2} + 2128f_{n+1} + 112f_n\right]$$
$$2511y_{n+5/3} - 2375y_{n+1} - 136y_n = \frac{5h}{3}\left[-5f_{n+2} + 640f_{n+3/2} + 430f_{n+1} + 21f_n\right]$$
$$7936y_{n+5/4} - 7625y_{n+1} - 311y_n = \frac{5h}{3}\left[-105f_{n+2} + 1040f_{n+3/2} + 4380f_{n+1} + 193f_n\right]$$
$$7936y_{n+7/4} - 7693y_{n+1} - 243y_n = \frac{21h}{3}\left[21f_{n+2} + 784f_{n+3/2} + 364f_{n+1} + 11f_n\right]$$
$$120y_{n+1} - 120y_n = \frac{h}{4}\left[106f_{n+2} + 1664f_{n+3/2} - 2511f_{n+4/3} + 1304f_{n+1} + 129f_n\right] \quad (17)$$

Definition (1.1)

A linear multistep method (LMM) is order P if $c_0 = c_1 = \cdots = c_{p-1}$ and $c_{p+1} \neq 0$ and is c_{p+1} called the error constant.

Definition (1.2)

A Linear Multistep Method (LMM) is consistent if it has order p ≥ 1

Definition (1.3): A-Stable (Dahlquist [6])

A numerical method is said to be A-stable if its region of absolute stability contains, the whole of the left-hand half plane $Reh\lambda < 0$

Definition (1.4): A $(\propto) -$ stable (Widlund [17])

A numerical method is said to be A $(\propto)$ stable, $\propto \epsilon\left(0, \pi/2\right)$, if it region is absolute stability contains the infinite wedge $W_\propto = [h\lambda - \propto < \pi - argh\lambda]$, it is said to be A(0)-stable if it is A $(\propto)$ - stable for some (sufficiently small) $\propto \epsilon\left(0, \pi/2\right)$

Definition (1.5)

A block method is zero –stable provided the root $\lambda_j, j = 1(1)s$ of the first characteristic polynomial $\rho(\lambda)$ specified as $\rho(\lambda) = det\left|\sum_{i=0}^{s} A^{(1)}\lambda^{(s-i)}\right| = 0$ satistfies $|\lambda_j| \leq 1$and for those roots with $|\lambda_j| = 1$, the multiplicity must not exceed two. The principal root of $\rho(\lambda)$ is denoted by $\lambda_1 = \lambda_2 = 1$.

Equation (17) constitute the member of a zero-stable block integrators of order $(5, 5, 5, 5, 5, 5, 5)^T$ with $C_6 = C_{p+1} = \left(-\frac{1}{180}, \frac{21}{320}, \frac{97}{405}, \frac{65}{324}, \frac{1135}{2304}, \frac{147}{1280}, \frac{17}{72},\right)$. The application of the block integrators with n = o give the values of y_1 and y_2 as shown in table (1-2).

RESULTS AND DISCUSSION

Recall, that it is a desirable property for a numerical integrator to produce solution that behave similar to the theoretical solution to a given problem at all times. Thus several definitions, which call for the method to possess some adequate region of absolute stability, can be found in several literatures. See Lambert [12]. Fatunla [7, 8, 9] etc. Following Fatunla

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Vol. 4 No. 4 July 2013

[8], the seven integrator proposed in this paper in equation (17) is put in the matrix-equation form and for easy analysis the result was normalized to obtain:

$$
\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{pmatrix}
\begin{pmatrix}y_{n+1}\\y_{n+5/4}\\y_{n+4/3}\\y_{n+3/2}\\y_{n+5/3}\\y_{n+7/4}\\y_{n+2}\end{pmatrix}
=
\begin{pmatrix}0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\end{pmatrix}
\begin{pmatrix}y_{n-7/4}\\y_{n-5/3}\\y_{n-3/2}\\y_{n-4/3}\\y_{n-5/4}\\y_{n-1}\\y_{n}\end{pmatrix}
+
$$

$$
\begin{pmatrix}
\frac{163}{60}&0&\frac{-837}{160}&\frac{52}{15}&0&0&\frac{-53}{240}\\
\frac{8724}{3072}&0&\frac{-41175}{8192}&\frac{325}{96}&0&0&\frac{-2675}{12288}\\
\frac{128}{45}&0&\frac{-74}{15}&\frac{4096}{1215}&0&0&\frac{-88}{405}\\
\frac{909}{320}&0&\frac{-12393}{2560}&\frac{69}{20}&0&0&\frac{-279}{1280}\\
\frac{925}{324}&0&\frac{-475}{96}&\frac{100}{27}&0&0&\frac{-275}{1296}\\
\frac{44149}{15360}&0&\frac{-207711}{40960}&\frac{931}{240}&0&0&\frac{-12299}{61440}\\
\frac{44}{15}&0&\frac{-27}{5}&\frac{64}{15}&0&0&\frac{-1}{15}
\end{pmatrix}
\begin{pmatrix}f_{n+1}\\f_{n+5/4}\\f_{n+4/3}\\f_{n+3/2}\\f_{n+5/3}\\f_{n+7/4}\\f_{n+2}\end{pmatrix}
$$

$$
\begin{pmatrix}
0&0&0&0&0&0&\frac{43}{160}\\
0&0&0&0&0&0&\frac{6595}{24576}\\
0&0&0&0&0&0&\frac{326}{1215}\\
0&0&0&0&0&0&\frac{687}{2560}\\
0&0&0&0&0&0&\frac{695}{2592}\\
0&0&0&0&0&0&\frac{10969}{40960}\\
0&0&0&0&0&0&\frac{4}{15}
\end{pmatrix}
\begin{pmatrix}f_{n-7/4}\\f_{n-5/3}\\f_{n-3/2}\\f_{n-4/3}\\f_{n-5/4}\\f_{n-1}\\f_{n}\end{pmatrix}
\quad (18)
$$

The first characteristics polynomial of the proposed 1-block 7 point method is

$$\rho(R) = \det\left|RA^{(0)} - A^{(1)}\right|$$

$$= \det\left\{ R \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} R & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & R & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{pmatrix}$$

$$= [\,R^6(R-1)] \tag{19}$$

$$\Rightarrow R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 0\,, R_7 = 1$$

From definition (1.5) and equation (19) the 1-block 7-point method is zero-stable and is also consistent (definition 1.2) as its order $(5, 5, 5, 5, 5, 5, 5)^T > 1$, then convergent following Henrici [11]

Stability analysis of the proposed method

Using the matlab program, we were able to plot the stability region of the proposed block method. This is done by reformulating a block method as general linear method to obtain the values of the matrices, A, B, U. V which are then substituted into stability matrix and stability function. Then the utilized maple program yields the stability polynomial of the block method.

We obtained the following values for A, B, U and V as:

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{129}{480} & \frac{1304}{480} & 0 & \frac{-2511}{480} & \frac{1664}{480} & 0 & 0 & \frac{-106}{480} \\ \frac{965}{95232} & \frac{1825}{7936} & 0 & 0 & \frac{325}{5952} & 0 & 0 & \frac{-176}{31744} \\ \frac{112}{7533} & \frac{2128}{7533} & 0 & 0 & \frac{526}{2511} & 0 & 0 & \frac{-68}{7533} \\ \frac{39}{1984} & \frac{81}{248} & 0 & 0 & \frac{15}{62} & 0 & 0 & \frac{-27}{1984} \\ \frac{35}{2511} & \frac{2150}{7533} & 0 & 0 & \frac{3200}{7533} & 0 & 0 & \frac{-25}{7533} \\ \frac{231}{31744} & \frac{1911}{7936} & 0 & 0 & \frac{1029}{1984} & 0 & 0 & \frac{441}{31744} \\ \frac{-1}{93} & \frac{4}{31} & 0 & 0 & \frac{63}{93} & 0 & 0 & \frac{5}{31} \end{pmatrix}$$

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Vol. 4 No. 4 July 2013

$$B=\begin{pmatrix} \frac{-1}{93} & \frac{4}{31} & 0 & 0 & \frac{64}{93} & 0 & 0 & \frac{5}{31} \\ \frac{129}{480} & \frac{1304}{480} & 0 & \frac{-2511}{480} & \frac{1664}{480} & 0 & 0 & \frac{-106}{480} \end{pmatrix}$$

$$U=\begin{pmatrix} 0 & 1 \\ 0 & 1 \\ \frac{7625}{7936} & \frac{311}{7936} \\ \frac{2368}{2511} & \frac{134}{2511} \\ \frac{459}{496} & \frac{37}{496} \\ \frac{2375}{2511} & \frac{136}{2511} \\ \frac{7693}{7936} & \frac{243}{7936} \\ \frac{32}{31} & \frac{-1}{31} \end{pmatrix}$$

$$V=\begin{pmatrix} \frac{32}{31} & \frac{-1}{31} \\ 0 & 1 \end{pmatrix}$$

Using a matlab program, we plot the absolute stability region of the proposed 1-block seven steps hybrid block Butcher's method.

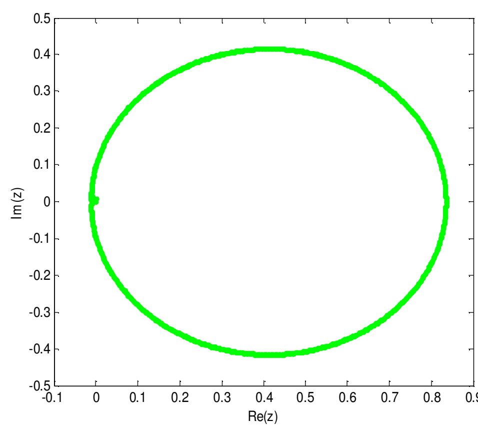

Figure 1

From definition (1.4) and figure (1) above, the proposed method (18) is A (α)-stable.

Numerical experiment

To illustrate the potentials of the new hybrid method constructed in this paper, we consider the initial value problem.

$y' = y \;, 0 \le x \le 2 \; y(0) = 1, \; h = 0.1$

Exact solution $y(x) = e^{-x}$

Table 1. Comparism of our block hybrid method with exact solution

N	*X*	*Exact solution*	*Proposed our method (18)*
0	0	1.0000000000	1.0000000000
1	0.1	0.9048374180	0.9048374164
2	0.2	0.8187307531	0.8187307517
3	0.3	0.7408182207	0.7408182181
4	0.4	0.6703200460	0.6703200438
5	0.5	0.6065306597	0.6065306566
6	0.6	0.588116361	0.5488116333
7	0.7	0.4965853038	0.4965853004
8	0.8	0.4493289641	0.4493298611
9	0.9	0.4065696597	0.4065696562
10	1.0	0.3678794412	0.3678794381

Table 2. Absolute errors of the problem

X	*Errors*
0	0.000E-00
0.1	1.600E-09
0.2	1.400E-09
0.3	2.600E-09
0.4	2.200E-09
0.5	3.100E-09
0.6	2.800E-09
0.7	3.400E-09
0.8	3.000E-09
0.9	3.500E-09
1.0	3.100E-09

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Vol. 4 No. 4 July 2013

CONCLUSION

A continuous block hybrid formula with one off-step point has been proposed and implemented as a self starting method in block form for the solution of first order ode. The convergent and stability properties of our method therefore, make it attractive for numerical solution of stiff and non-stiff problems. We have demonstrated the accuracy of the block method by applying it on a numerical problem.

REFERENCES

[1] Akinfenwa, O. A., Jator, S. N. & Yao, N. M. (2011). A Linear Multistep Hybrid Methods with continuous coefficient for Solving Stiff Ordinary Differential Equation. *Journal of Modern Mathematics and Statistics, 5*(2), 47-53.

[2] Atkinson, K. E. (1987). *An introduction to Numerical Analysis* (2nd Edition). New York: John Wiley and sons.

[3] Awoyemi, D. O. (1992). *On some continuous Linear Multistep Methods for Initial Value Problems* ,PhD.Thesis (Unpublished), University of Ilorin,Nigeria.

[4] Butcher, J. C. (1965). A modified multistep method for the numerical integration of ordinary differential equations. *J. Assoc. Comput. Math.,* 12, pp.124-135

[5] Butcher, J. C. (2003). *Numerical Methods for Ordinary differential systems*. West Sussex,England: John Wiley & sons.

[6] Dahlquist, G. (1963). *A special stability problem for linear multistep methods*, BIT, 3, pp.27-43

[7] Fatunla, S. O. (1991). Block methods for second order IVP's. *Inter.J.Comp.Maths, 72*(1).

[8] Fatunla, S. O. (1992). *Parallel methods for second order ODE's Computational ordinary differential equations*.

[9] Fatunla, S. O. (1994). Higher order parallel methods for second order ODE's.Scientific Computing. *Proceeding of fifth International Conference on Scientific Computing*.

[10] Gear, C. W. (1965).Hybrid methods for initial value problems in ordinary differential systems. *SIAM J. Numer. Anal.,* 2, pp.69-86.

[11] Henrici, P. (1962). *Discrete variable methods for ODE's*. New York: John Wiley.

[12] Lambert, J. D. (1973). *Computational methods for ordinary differential equations*. New York: John Wiley.

[13] Lambert, J. D. (1991). *Numerical methods for ordinary differential systems*. New York: John Wiley.

[14] Lie, I. & Norset, S. P. (1989). Super Convergence for Multistep Collocation. *Math. Comp., 52*.

[15] Onimanyi, P., Awoyemi, D. O., Jator, S. N. & Sirisena, U. W. (1994). New Linear Multistep Methods with continuous coefficients for first order initial value problems. *J.Nig.Math.Soc.*

[16] Onumanyi, P., Sirisena, U. W. & Jator, S. N. (1999). Continuous finite difference approximations for solving differential equations. *Inter.J.Comp.Maths*.

[17] Widlund, O. B. (1967). *Notes on unconditional stable linear multistep methods*, BIT 7, pp. 65-70

[18] Yusuph, Y. & Onumanyi, P. (2002). New Multiple FDM's through Multistep Collocation for Special Second order ODE's. *The Journal of the Mathematical Association of Nigeria , 29*(2).

[19] Yusuph, Y. & Onumanyi, P. (2002). Improved Fatunla Block Method. *The Journal of the Mathematical Association of Nigeria, 29*(2).